The Semantics of Indicative Conditionals: The Return of the Trivalent Knights

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COME LA GENTE IMMAGINA IL PIEMONTE
COME È REALMENTE

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Notabene: we focus on prediction-oriented conditionals and leave out degenerate cases such as Dutchman conditionals.

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Overall Aim: Defending the truth-functional view.

I. Bivalent Semantics

The Classical Truth-Functional Account

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The truth conditions of the conditional "if A, then C" are equivalent to those of the material conditional $A \supset C$.

A	C	$A \rightarrow C$
T	T	T
T	F	F
F	T	T
F	F	T

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Paradoxes of Material Implication

The truth value of the material conditional is not sensitive to the connection between antecedent and consequent.

- "If it rains in Vercelli on September 3, 2018, then many people will attend the FINO conference."
- "If the sun shines in Turin on September 3, 2018, then the second day of FINO will take place in Novara."

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Stalnaker Semantics

Consider a possible world in which A is true and otherwise differs minimally from the actual world. "$A \rightarrow C$" is true (false) just in case C is true (false) in that possible world. (Stalnaker 1968, 33–34)

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T	F	F
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For the record, this is different from

Adams's Thesis

$$Ass(\mathrm{A} \to \mathrm{C}) = p(\mathrm{C}|\mathrm{A}) \qquad \text{(Adams's Thesis (AT))}$$

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The Import-Export Principle

The following two principles are equivalent:

1. If A and B, then C.
2. If A, then (if B, then C).

Gibbard's Triviality Result

In a famous paper from 1981, Gibbard showed that all conditionals $A \to C$ with the following conditions:

- It is at least as strong as the material conditional. $(A \to C \Rightarrow A \supset C.)$
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Vassalage (Jackson): Truth conditions $\sim$ material conditional. Disentangled from the assertability of a conditional.

II. Trivalent Semantics: The de Finetti Conditional

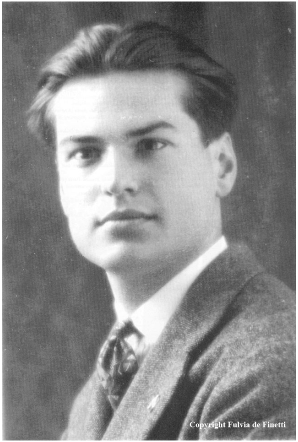

Figure: Two pioneers of the epistemology of conditionals: Quine and De Finetti.

The Trivalent Account: History

The trivalent account assimilates indicative conditionals to conditional predictions/assertions.

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There are similar passages in Adams 1965 ("The Logic of Conditionals") regarding conditional bets, but neither makes much out of this observation.

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- Truth conditions for conditionals ∼ conditions for settling a conditional bet
- Conditional recognized as true = both the antecedent and the consequent have been verified

A Template for Trivalent Semantics

$f_{\rightarrow}$	1	0
1	1	0
0	#	#

$f_{\rightarrow}$	1	$1/2$	0
1	1	.	0
$1/2$	.	.	.
0	$1/2$	.	$1/2$

Figure: "Defective" two-valued truth table (left) and incomplete three-valued expansion (right) for the conditional functor $f_{\rightarrow}$.

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$$\begin{aligned} Ass(A \to C) &= p(A \to C \text{ is true} | A \to C \text{ has a classical truth value}) \\ &= p(A \wedge C | A) \\ &= p(C|A) \end{aligned} \qquad (1)$$

Adams' Thesis is a simple consequence of the trivalent approach!

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Project: Find the most convincing combination of these three parameters! (More challenging than for bivalent logic.)

Standard Logical Operations

All standard logical operators are interpreted via the Strong Kleene truth table.

	$f_\neg$
1	0
$1/2$	$1/2$
0	1

$f_\wedge$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$1/2$	$1/2$	0
0	0	0	0

Analogous for disjunction.

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$f_{\to_{DF}}$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$1/2$	$1/2$	$1/2$
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Table: The truth table for the de Finetti conditional.

Evaluations and Truth

Classical/Strong Kleene/de Finetti Evaluations

Let $\mathcal{L}$ be a first-order propositional language and $\mathcal{L}_{\rightarrow}$ be the extended language with the conditional connective '$\rightarrow$'.

- A classical evaluation is a function $v : \mathcal{L} \rightarrow \{1, 0\}$ that interprets '$\neg$' and '$\wedge$' by the functors $f_{\neg}$ and $f_{\wedge}$ restricted to the values 1 and 0.
- A strong Kleene (SK-) evaluation is a function $v : \mathcal{L} \rightarrow \{1, 1/2, 0\}$ that interprets '$\neg$' and '$\wedge$' by the functors $f_{\neg}$ and $f_{\wedge}$.
- A de Finetti (DF-) evaluation is a function $v : \mathcal{L}_{\rightarrow} \rightarrow \{1, 1/2, 0\}$ that interprets '$\neg$', '$\wedge$', and '$\rightarrow$' by the functors $f_{\neg}$, $f_{\wedge}$ and $f_{\rightarrow_{DF}}$.

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- An evaluation $v : \mathcal{L}_{\rightarrow} \rightarrow \{1, 1/2, 0\}$ makes a sentence A tolerantly true (or T-true) provided $v(A) > 0$.

Validity Notions

Validity

Given an evaluation for the sentences of $\mathcal{L}$ (respectively $\mathcal{L}_{\rightarrow}$), we say that:

- $\Gamma \models_{SS} A$ if every evaluation that makes all sentences of Γ S-true also makes A S-true.

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Question: Which validity relation is the most appropriate one for the de Finetti conditional $f_{\rightarrow_{DF}}$?

Problems with Validity

SS- and ST-validity Allows for conjunction introduction:

$$A \rightarrow C \models_{SS} A \wedge C$$

and implication to the converse:

$$A \rightarrow C \models_{SS} C \rightarrow A$$

Sentential validities and the Identity Law ($\models_{SS} A \rightarrow A$) also fail.

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Sentential validities and the Identity Law ($\models_{SS} A \to A$) also fail.

TT-validity Invalidates the above inferences and preserves sentential validities, but modus ponens fails:

$$A, A \to C \not\models_{TT} C$$

Problems with Validity

SS- and ST-validity Allows for conjunction introduction:

$$A \to C \models_{SS} A \wedge C$$

and implication to the converse:

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(SS ∩ TT)- and TS-validity Inherits the problems of SS- and TT-validity.

The Trilemma for the de Finetti Conditional

There is no fully satisfactory validity relation for the de Finetti conditional:

The Validity Trilemma

Irrespective of whether SS, TT, ST, TS or SS $\cap$ TT is chosen for validity, the DF-conditional either must

1. fail Modus Ponens;
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	Modus Ponens	Identity/Sent. Validities	$A \to C \models C \to A$
Ideal case	✓	✓	×
SS	✓	×	✓
TT	×	✓	×
ST	✓	✓	✓
TS	×	×	×
SS ∩ TT	×	×	×

TT-validity and the Deduction Theorem

Of all candidates, TT-validity seems to be the least evil (note that MP holds for *classical* formulas). It also validates

Commutation with Negation

$$\neg(A \rightarrow C) \equiv_{\mathsf{TT}} A \rightarrow \neg C$$

(In agreement with our use of indicative conditionals in ordinary language.)

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Conditional Introduction

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Conditional Introduction

$$\Gamma, A \models_{TT} C \;\Rightarrow\; \Gamma \models_{TT} A \to C$$

Failure of Deduction Theorem

$$\Gamma \models_{TT} A \to C \;\not\Rightarrow\; \Gamma, A \models_{TT} C$$

(Counterexample: $\Gamma = \emptyset$, $A = 1/2$, $C = 0$.)

Intermediate Conclusions

Advantages and drawbacks of the de Finetti conditional:

- Principled, intuitive and well-motivated semantics.
  ($\rightarrow$ supposition of the antecedent, conditional bets) for the trivalent conditional.
- Solves major problems of bivalent semantics.
  ($\rightarrow$ Import-Export, connection to epistemology, etc.)
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  $\to$ Validity Trilemma

New Research Question: solve problem by modifying (the second row of) de Finetti's truth table.

III. Trivalent Semantics: Jeffrey Conditionals

A Template for Trivalent Semantics

Starting point of trivalent semantics:

$f_\rightarrow$	1	1/2	0
1	1	.	0
1/2	.	.	.
0	1/2	.	1/2

This template can be filled in in various ways. Two proposals from the literature:

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This template can be filled in in various ways. Two proposals from the literature:

$f_{\to C}$	1	1/2	0
1	1	1/2	0
1/2	1	1/2	0
0	1/2	1/2	1/2

$f_{\to F}$	1	1/2	0
1	1	1/2	0
1/2	1/2	1/2	0
0	1/2	1/2	1/2

Figure: Truth tables for the Cooper conditional (1968, left) and the Farrell conditional (1979/86, right).

Cooper and Farrell Evaluations

Cooper and Farrell Evaluations

- A Cooper evaluation (or C-evaluation) is a function $v : \mathcal{L}_{\rightarrow} \rightarrow \{1, {}^1\!/_2, 0\}$ interpreting '$\neg$', '$\wedge$', and '$\rightarrow$' by the functors $f_{\neg}$, $f_{\wedge}$ and $f_{\rightarrow_C}$.

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- A Farrell evaluation (or F-evaluation) is a function $v : \mathcal{L}_{\rightarrow} \rightarrow \{1, {}^1/_2, 0\}$ interpreting '$\neg$', '$\wedge$', and '$\rightarrow$' by the functors $f_{\neg}$, $f_{\wedge}$ and $f_{\rightarrow_F}$.

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1. Which truth table should we choose?
2. What is the appropriate validity relation?

Cooper: TT-validity natural; indeterminate antecedents $\sim$ true antecedents.

Jeffrey Conditionals

Axiomatic Requirement (Jeffrey, 1963): we focus on truth tables that satisfy Modus Ponens under a TT-validity relation.

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Jeffrey Conditionals

A Jeffrey conditional is any binary three-valued operator of the form:

$f_\rightarrow$	1	1/2	0
1	1	d_1	0
1/2	d_2	d_3	0
0	1/2	d_4	1/2

where $d_i \in \{1/2, 1\}$ for $1 \leq i \leq 4$.

Like the de Finetti conditional, Jeffrey conditionals recover Adams's Thesis ($\text{Ass}(A \rightarrow C) = p(C|A)$) for classical propositions $A, C \in \mathcal{L}$.

Validity: Deduction Theorem and Trilemma Resolution

Deduction Theorem

- Any Jeffrey conditional TT-validates the full Deduction Theorem: $\Gamma, A \models_{TT} C$ if and only if $\Gamma \models_{TT} A \to C$.
- No Jeffrey conditional validates the full deduction theorem for SS-, $TT \cap SS$, ST and TS-validity.

This result singles out TT-validity as a privileged validity relation for the class of Jeffrey conditionals.

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Trilemma Resolution and Failure of Contraposition

Under a TT-notion of validity, any Jeffrey conditional

- satisfies Modus Ponens and the Identity Law;
- invalidates the entailment from $A \to C$ to $A \& C$ and $C \to A$.

Truth Tables: Interaction with (Strong Kleene) Negation

Jeffrey Conditionals and Commutation with Negation

Among all Jeffrey conditionals, only Cooper's validates the full commutation schema for negation.

$$\neg(A \to C) \equiv_{TT} A \to \neg C$$

The Cooper conditional is also the most "natural" truth table among all Jeffrey conditionals (indeterminate antecedents $\sim$ true antecedents).

Intermediate Conclusions

De Finetti conditionals run into the validity trilemma. The Jeffrey Conditionals with the TT-validity relation...

- block this trilemma;
- validate the full Deduction Theorem;
- support—in the Cooper variant—full commutation with negation.

The Cooper conditional strikes the best balance of logical, conceptual and epistemic properties.

What about the proof theory?

IV. Proof Theory and Algebraization

Overview of Proof Theory

Our proof theory has three parts:

Tableaux Calculus Soundness and completeness results for a tableaux calculus.
(Allows only for finite set of premises.)

Sequent Calculus Soundness and completeness results for a sequent calculus à la Gentzen.

Algebraization Construction of the Lindenbaum-Tarski algebra; canonical model theorem.

Sequent Calculus: Axioms and Rules

Axiom:

$$\frac{}{\Gamma, A \mid \Delta, A \mid \Sigma, A}\ \text{SRef}$$

Rules (excerpts):

$$\frac{\Gamma, A \mid \Delta \mid \Sigma}{\Gamma \mid \Delta \mid \Sigma, \neg A}\ \neg\text{-}0 \qquad \frac{\Gamma \mid \Delta, A \mid \Sigma}{\Gamma \mid \Delta, \neg A \mid \Sigma}\ \neg\text{-}{}^1\!/_2 \qquad \frac{\Gamma \mid \Delta \mid \Sigma, A}{\Gamma, \neg A \mid \Delta \mid \Sigma}\ \neg\text{-}1$$

$$\frac{\Gamma \mid \Delta, A \mid \Sigma, A \qquad \Gamma \mid \Delta \mid \Sigma, B}{\Gamma \mid \Delta \mid \Sigma, A \to B}\ \to\text{-}1$$

Satisfaction and Soundness

Satisfaction and Validity

A C-evaluation v satisfies a sequent $\Gamma \mid \Delta \mid \Sigma$ if:

- there is an $A \in \Gamma$ s.t. $v(A) = 0$, or
- there is a $B \in \Delta$ s.t. $v(B) = 1/2$, or
- there is a $C \in \Sigma$ s.t. $v(C) = 1$.

A sequent $\Gamma \mid \Delta \mid \Sigma$ is C-valid if it is satisfied by every C-evaluation.

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Lemma

For every sets of formulae Γ and Δ:

$$\Gamma \models_{CTT} \Delta \text{ if and only if } \Gamma \mid \Delta \mid \Delta \text{ is C-valid}$$

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Lemma

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Soundness Theorem for Sequent Calculus

If $\Gamma \vdash_{CTT} \Delta$, then $\Gamma \models_{CTT} \Delta$.

(Proof by induction on the length of the derivation $\Gamma \mid \Delta \mid \Delta$.)

Satisfaction and Completeness

Countermodels and Derivations

For every triple of sets of formulae Γ, Δ, and Σ, exactly one of the two following cases is given:

1. There is a derivation of $\Gamma \mid \Delta \mid \Sigma$ in CTT.
2. $\Gamma \mid \Delta \mid \Sigma$ has a countermodel.

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Completeness Theorem for Sequent Calculus

For every set Γ of formulae and every formula A:

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Completeness Theorem for Sequent Calculus

For every set Γ of formulae and every formula A:

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(Proof immediate from the previous result by contraposition:

$$\text{if } \Gamma \nvdash_{CTT} \Delta, \text{ then } \Gamma \not\models_{CTT} \Delta.)$$

Algebraization

Provable Equivalence

For every set of formulae Γ, define the relation of provable equivalence $\sim^c_\Gamma$ as follows:

$$A \sim^c_\Gamma B \text{ if and only if } \Gamma \vdash_{\mathsf{CTT}} A \leftrightarrow B$$

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Notabene: The equivalent construction for the de Finetti conditional does not induce an equivalence relation!

The Lindenbaum-Tarski Algebra

The Cooper-Lindenbaum-Tarski algebra of Γ is the structure

$$\mathcal{C}(\Gamma) = \langle \mathsf{For}(\mathcal{L}_{\rightarrow})/ \sim^{c}_{\Gamma}, \sqcap_{\Gamma}, \sqcup_{\Gamma}, -_{\Gamma}, \rhd_{\Gamma}, \mathbf{0}_{\Gamma}, \mathbf{1}_{\Gamma} \rangle$$

where:

$$[A]_{\Gamma} \sqcap_{\Gamma} [B]_{\Gamma} := [A \wedge B]_{\Gamma} \qquad [A]_{\Gamma} \sqcup_{\Gamma} [B]_{\Gamma} := [A \vee B]_{\Gamma}$$

$$-_{\Gamma}[A]_{\Gamma} := [\neg A]_{\Gamma} \qquad [A]_{\Gamma} \rhd_{\Gamma} [B]_{\Gamma} := [A \rightarrow B]_{\Gamma}$$

$$[\bot]_{\Gamma} := \mathbf{0}_{\Gamma} \qquad [\bot \rightarrow \top]_{\Gamma} := \mathbf{1/2}_{\Gamma}$$

$$[\top]_{\Gamma} := \mathbf{1}_{\Gamma}$$

The Canonical Model Theorem

Definition: canonical evaluations

Let a Γ-canonical evaluation be a function $c_\Gamma : \mathrm{For}(\mathcal{L}_\to) \longmapsto \mathcal{C}(\Gamma)$ such that for every propositional variable p,

$$c_\Gamma(p) := [p]_\Gamma$$

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Canonical model theorem

For every set $\{\Gamma, A\} \subseteq \mathrm{For}(\mathcal{L}_\to)$, the following claims are equivalent:

(i) $\Gamma \vdash_{\mathrm{CTT}} A$

(ii) $\Gamma \models_{\mathfrak{C}} A$ (A follows from Γ in all Cooper algebras)

(iii) $c_\Gamma(A) = \mathbf{1}_\Gamma$ or ${}^1\!/_2{}_\Gamma$

V. Summary

Bivalent Conditionals

There are three major views on the truth conditions of indicative conditionals in bivalent logic:

- The bivalent truth-functional view (e.g., the material conditional).
- The non-truth-functional view (e.g., possible worlds semantics).
- The suppositionalist view ($\rightarrow$ gappy TC, shift focus to probability/acceptability)

Each of these views has its advantages, but none of them is fully convincing.

Trivalent Conditionals

Set up trivalent semantics of conditionals where indicative conditionals correspond to conditional predictions.

- False antecedent leads to an indeterminate truth value.
- Fully analogous to de Finetti's idea of conditional bets.
- Best combination: Cooper's truth table for the conditional, and the TT-notion of validity
- Preserves inference principles such as Modus Ponens, Import-Export, Deduction Theorem, etc.
- Attractive proof theory and semantics
  1. Soundness and completeness proofs for tableaux and sequent calculi
  2. Algebraization and canonical model theorem (Lindenbaum-Tarski-Algebra)

Exploration: what about counterfactuals?

If we adopt the de Finettian semantics for counterfactuals, too, then we obtain that $A > C$ is always indeterminate.

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- Counterfactuals can never be true (or false).
  1. If switch A or B had been down, the light would be off. (Both switches are up and the light is on.)
  2. If we move down switch A or B, the light will be off. (Ditto.)
- How can one ever verify or falsify the first statement?

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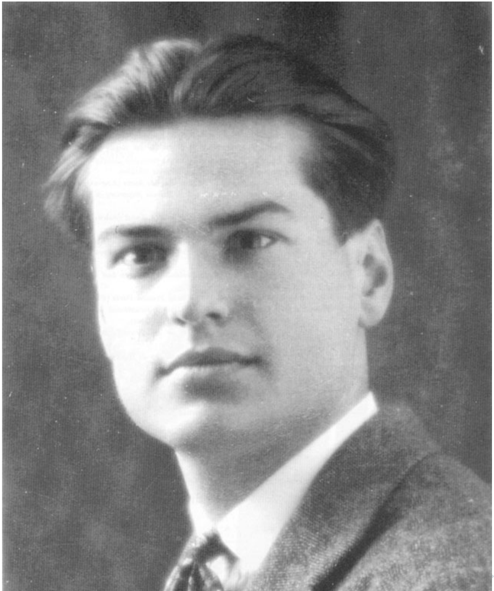
  "I reject questions of counterfactual form as either nonsense or as colorful ways of asking about conditional probabilities." (Jeffrey, 1991)

The Master of Trivalence



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